Manipulative Calculus: Active Learning with 3D Models



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Example from multivariable calculus

Parameterize the paraboloid $z = x^2 + y^2 + 1$. On your model of the paraboloid, sketch the grid curves defined by your parameterization.



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Question: How was working on this problem with the model different than it would have been without the model?

Example from multivariable calculus

An artist makes a vase shaped like the portion of the paraboloid $z = x^2 + y^2 + 1$ with $z \le 10$ (the axes are marked in inches). She spray paints the outside of the vase with gold paint so that the density of gold on the vase at (x, y, z) is f(x, y, z) = z mg per square inch. How much gold does she use in all?

Other multivariable topics

 Contour maps vs, graphs, partial derivatives, directional derivatives, optimization (constrained and unconstrained)



(Inspired by Raising Calculus to the Surface)

Other multivariable topics

Triple integrals





Other multivariable topics

Traces of surfaces, Stokes' Theorem



Practical considerations

- Designing and producing models
- Preparing instructors to facilitate



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CAs 50% of course assistants said they think they would have learned multivariable calculus better as a student if they had used 3D props in class (14% did not, 36% said maybe).

Coming soon...

Volumes of revolution lessons for Calc 2





For more info...

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