Building Course Embedded Undergraduate Research Experiences (CURE) in a Mathematics Major Pathway

Undergraduate Research Opportunities Center
UROC
California State University, Monterey Bay

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CSUMB Context

- 7,545 students (Fall 2018)
  - 62.8% are female
  - 52.7% underrepresented minority
  - 50.4% first generation
- 105 Math majors (Fall 2018)
  - 52.4% female
  - 42.9% underrepresented minority
  - 61.9% first generation

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Challenges with Undergraduate Research in Mathematics

There are many obstacles with undergraduate research in mathematics:

- Takes a lot of time for students to get research ready.
- Lack of motivation among students to engage in math research
- Lack of knowledge about discovery and relevance of math research

We along with Peri Shereen became CURE Fellows in the summer of 2018 to explore how to overcome these challenges using CURE in our courses.

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What is a CURE?

A CURE is a Course-Embedded Undergraduate Research Experience that must contain the following:

- The use of Scientific Practices
- Discovery
- Broadly Relevant or important work
- Collaboration
- Iteration (Building on prior knowledge, repeating, experiments).

CUREs enable more students to gain research experience, compared with the traditional research experiences, in which a student works with a faculty researcher in a specific topic of research.

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Benefits of a CURE

Many positive outcomes for students

- including increased engagement in learning the subject
- understanding of scientific/mathematical processes
- interest in science as a career
- persistence in mathematical and scientific fields.

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What is a "Math CURE"?

We developed the objectives of a Math CURE that helped us to define a Math CURE.

1. **Students participate in mathematical research by**
   a. Generating research questions
   b. Developing conjectures
   c. Proving or disproving their conjectures
   d. Presenting results
2. **Students make discoveries (unique to themselves) in content that is not part of the current curriculum. Students will investigate their research by**
   a. Calculating (counter)examples
   b. Searching for patterns
   c. Making meaning of their examples
3. **Students develop their own identity within the broader mathematical community by**
   a. Exploring topics mathematicians are currently investigating
   b. Researching within and with a community of their peers
   c. Summative project or presentation

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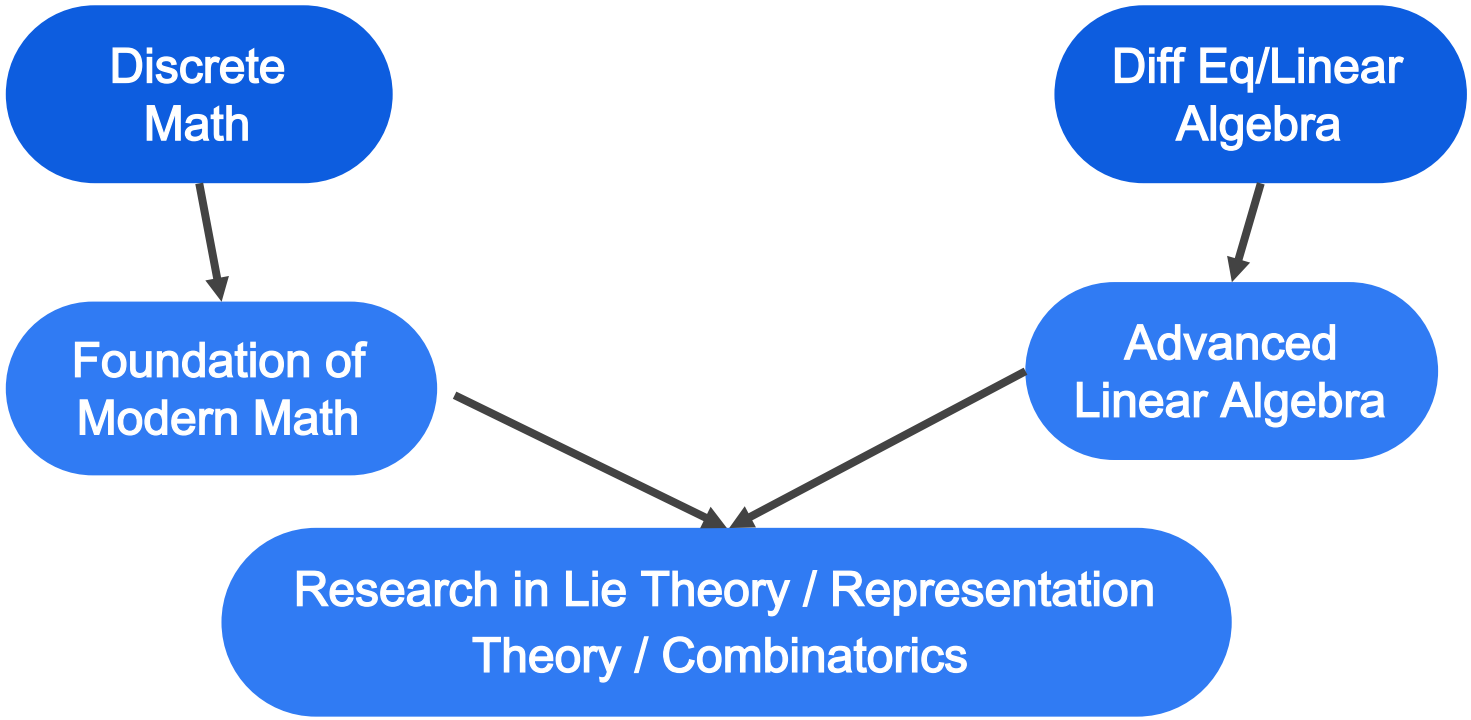
A CURE Pathway for Math Major

We and Peri Shereen developed each CURE with the intent of developing the following pathway:



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What we did with CURE so far..

Math 170 (Discrete Math, Deka):

- Piloted once, implemented once and going for the second implementation in Fall 2019

Math 265 (Diff eq/Lin Al, Wand):

- Piloted once and going for the first implementation in Fall 2019

Math 322 (Foundations of Math, Shereen):

- Piloted once and going for the first implementation in Fall 2019

Math 265: Differential equation & Linear Algebra CURE

- This CURE was semester long **optional** project with 5 exercises and checkpoints that mirrored the content from the course
- The 15 out of 25 Students who participated got into teams of 3-4

- Each team was allowed to submit one draft of each exercise for feedback

- Drafts and final product were typed using LaTex

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Exercises and Course Content

01	Group Theory	• Pre-Vector Spaces
02	Lie Algebras	• Post Vector Spaces
03	Lie Algebra Structure and Subspace of 2X2 matrices	• Subspaces
04	Representation Theory and Basis of 2X2 traceless matrices	• Basis
05	Weight Spaces	• Eigenvalues and Eigenvectors

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Checkpoints and Meetings

First Draft due	Content
10/12	Problem 1
10/19	Problem 2
10/26	Problem 3
11/2	Problem 4
12/14	Problem 5

- Teams had 1 week to submit corrections
- Teams were required to meet with after problem 1 to discuss progress and could make appointments with me as necessary afterwards

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Connection To CURE Goals

In developing the exercises the goals I kept in mind:

1. Students *participate* in mathematical research by
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   b. Developing conjectures
   c. Proving or disproving their conjectures
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2. Students *make discoveries* (unique to themselves) in content that is not part of the current curriculum. Students will investigate their research by
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Math 170: Discrete Mathematics

- A freshman course required for both mathematics and computer science majors with a prerequisite of Precalculus or equivalent
- Builds the foundation for mathematical language and valid argumentation along with introduction to various discrete structures
- A prerequisite for all math majors before getting to the proof course which is a foundation for all upper division math courses.
- Usually 2/3 sections/semester with 35-40 students

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Math 170 CURE project

- A semester long group project with three parts that runs parallel to the course
- Each project part takes students through the various stages of the process of conducting research in mathematical sciences
- Each part has a deliverable along with a final paper and a PowerPoint presentation to summarize their journey at the end
- Students work in groups of 3-4

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Project part 1
Formulating a research question through investigation

Task: In this part each group is **given a mathematical word(s) to research and form research questions about the word(s).** A set of guidelines are given to follow. Words are new but connected to the course content.

Objective: To help student learn how to use library and other tools to search for existing body of research relevant to discrete structures in mathematics.

Deliverable: Summarize findings as an article (typed up).

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Project part 2

Looking for patterns and formulating a conjecture

Task: Each group will be analyzing examples and counterexamples to observe patterns for one of the research questions they came up with in Part 1 and then form a conjecture to answer their question

Objective: To learn the process of formulating a conjecture to answer your research question

Deliverable: Summarize findings as an article (typed up)

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Project part 3
Proving or disproving a conjecture

Task: Investigate in your group how to prove or disprove your conjecture and provide a proof of the conjecture if you can or disprove the conjecture or discuss challenges you face to complete either.

Objective: To learn how to identify resources to justify and strengthen your conjecture, how to identify and demonstrate appropriate proof methodologies

Deliverable: Summarize findings as an article (typed up) and do a powerpoint presentation in class

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Data Results

Survey created by faculty including constructions developed by:

1. **URSSA** (undergraduate research student self-assessment)
   a. URSSA is an online survey instrument for use in evaluating student outcomes of undergraduate research experiences in the sciences.
2. **LCAS** (laboratory course assessment survey)

For our MATH courses we administered a similar, but modified survey to incorporate mathematical context.

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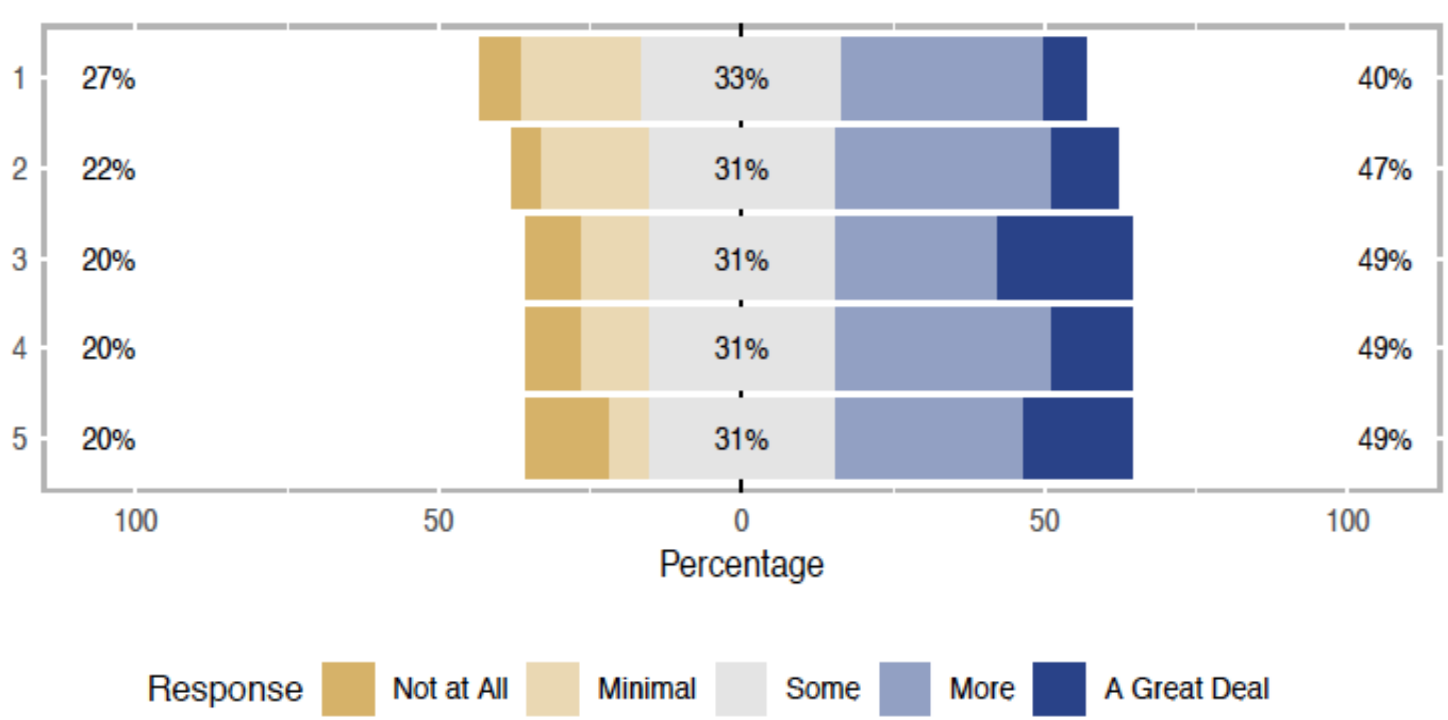
Self-Efficacy

How much did you GAIN in the following areas as a result of your most recent research experience?



1. Confidence in my ability to contribute to mathematics
2. Comfort in discussing mathematics concepts with others
3. Comfort in working collaboratively with others
4. Confidence in my ability to do well in future mathematics courses
5. Ability to work independently

Progress on comfort on working with others and ability to work independently!

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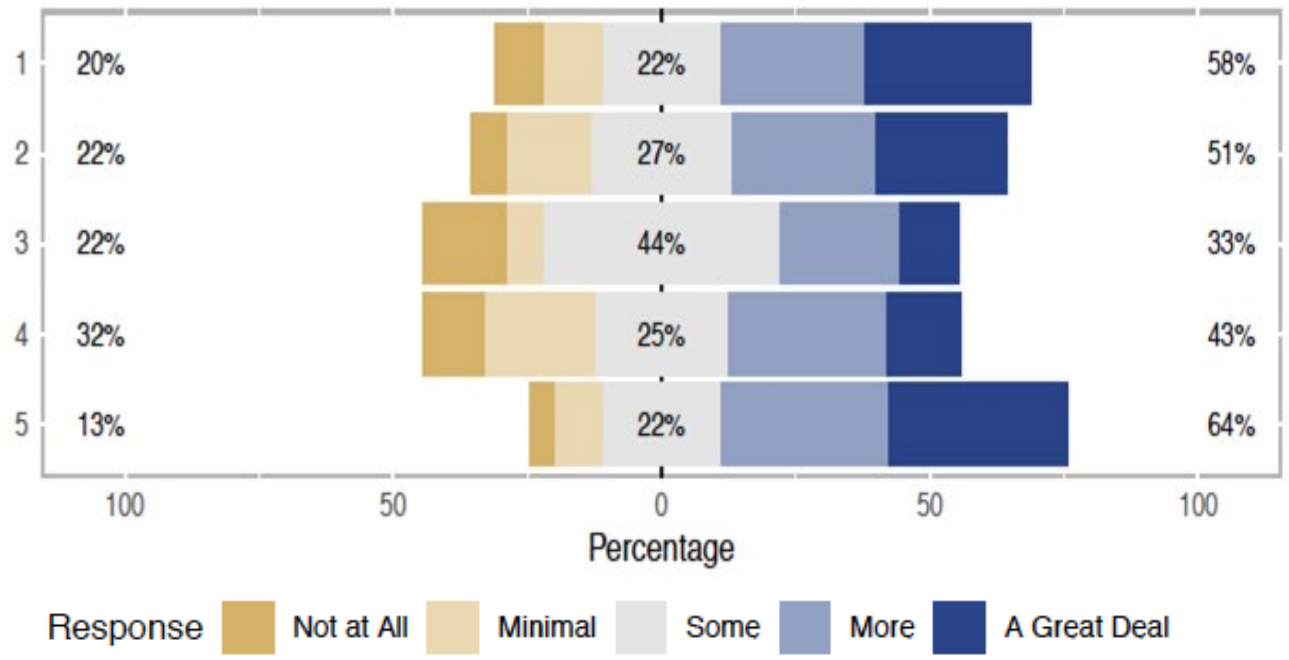
Motivation

Compared to BEFORE doing your most recent research, HOW LIKELY ARE YOU NOW to agree with the statement:



1. Learning mathematics is interesting
2. I am curious about discoveries in mathematics
3. The mathematics I learn is relevant to my life
4. Learning mathematics makes my life more meaningful
5. Learning mathematics will help me get a good job

Progress on motivation by increasing interest in learning, curiosity in discovery and job in math

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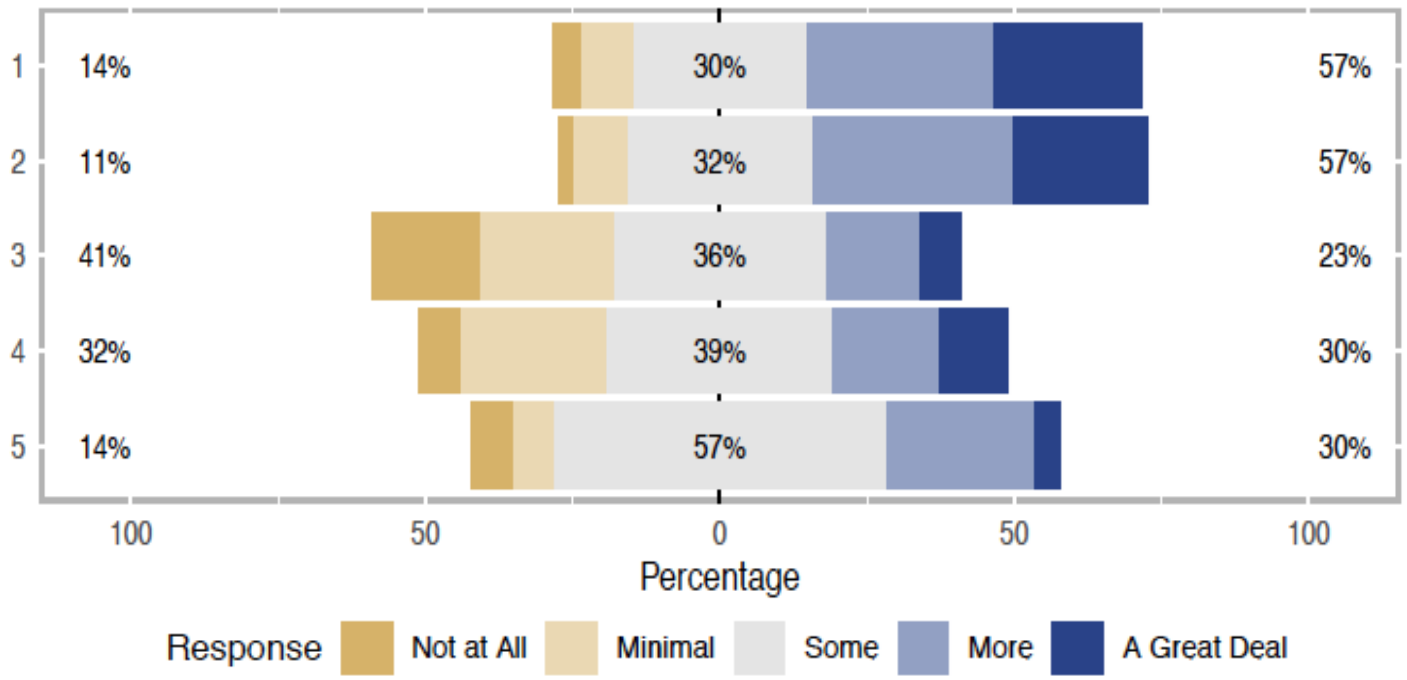
Collaboration in Class

During this most recent research project, I was encouraged to:



1. Discuss elements of my investigation with classmates or instructors
2. Reflect on what I was learning
3. Contribute my ideas and suggestions during class discussions
4. Help other students collect or analyze data
5. Provide constructive criticism to classmates and challenge each other's interpretations

Students felt encouraged to discuss their investigation with peers and instructor and reflect in learning

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Math 170 Comparison of 1st and 2nd implementation

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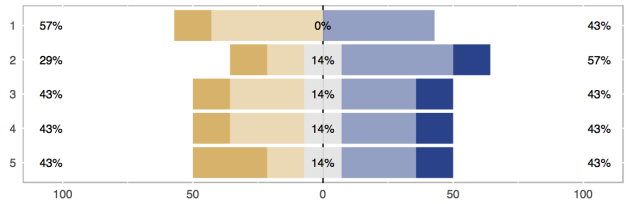

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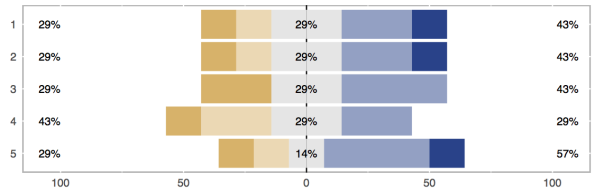
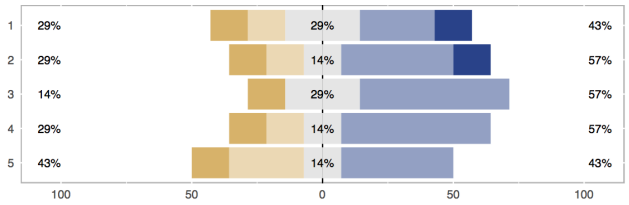


Discovery & Relevance

During this most recent research project, I was expected to:



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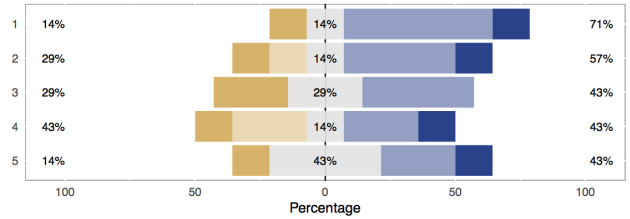

2018

Self-Efficacy

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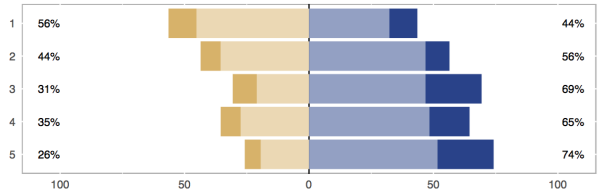

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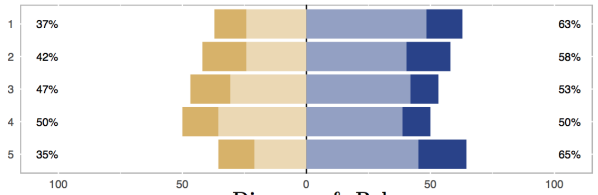

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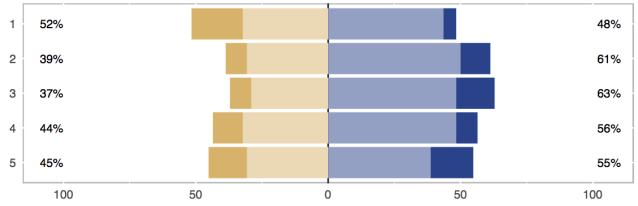

Collaboration in Class

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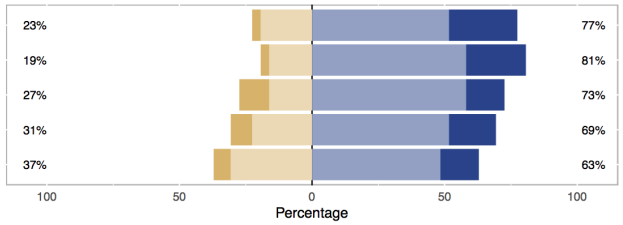

2019

Student Reflections

Math 170

- The research project we did was good because it gave us experience working in a team and also showed us the process used when doing mathematical research.
- The cure project was interesting and it was fun because it changed the routine of just taking notes in class.
- I didn't know about math research before this
- It was challenging, but made math fun
- I want to do more math research
- I know that I am not a math person

Math 265

- I feel a lot stronger in the force (math) after doing the CURE
- My group members were not very cooperative except one person. I definitely feel as though I can and would like to commit more within being more comfortable with explaining mathematics above the Calculus I realm.
- I already wanted to get my masters and credentials before this. I already planned on teaching high school and community college.

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Instructor's reflections

- Be flexible to each group's process.
- It's ok if the final product has unanswered questions, that's research.
- Meeting with each group was very important to the process, would consider meeting more often.
- It is demanding of your time.
- In the future, I would be more explicit with expectations.
- Need to be more clear about expectations of their summaries.
- Need to have frequent discussion about project in class to keep them on track

What we would do Differently

Math 265:

1. Require more check-in meetings with instructor. (Maybe…)
2. Make the CURE non optional.
3. Develop clearer expectations of final product.
4. Less scaffolding of the Math, more for paper.
5. Making clear the importance of citation.

Math 170:

1. More discussion in class about expectation
2. More team building activity in class
3. Provide more feedback on each part and find ways to do it better
4. Have more check-in among instructors so they have the support to help the students
5. Look for resources to help with writing

Resources

1. Auchincloss, Lisa Corwin, Laursen, Sandra L., Branchaw, Janet L., Eagan, Kevin, Graham, Mark, Hanauer, David I., . . . Dolan, Erin L. (2014). *Assessment of Course-Based Undergraduate Research Experiences: A Meeting Report*. CBE - Life Sciences Education, 13(1), 29-Life Sciences Education, 2014, Vol.13(1), p.29-40.
2. Halmos, P. (1991). *Problems for mathematicians, young and old* (Dolciani mathematical expositions ; no. 12). Washington, D.C.]: Mathematical Association of America. (Problem 4H)
3. Sloane, N. (2003). *The On-Line Encyclopedia of Integer Sequences*. Notices American Math. Soc., Vol. 50 (Sept. 2003), 912-915.
4. http://www.asbmb.org/asbmbtoday/201604/Education/CURES/

Thank you!

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Extra Slides for Math 322

Foundations of Modern Math

- ★ Introduction to proof course for our majors
- ★ Prerequisites
  - Discrete Mathematics
  - Calculus 2
- ★ Class size
  - Typically 15-25 students
  - Fall 2018: 24 students
- ★ Fall 2018 43% of our majors were upper division transfer students.

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Class Portfolio

1. Proof Portfolio
    a. 6 proof problems
    b. 2 rounds of drafts
2. Discovery Portfolio
    a. CURE project
    b. Final product: group poster and individual summary
3. Professional Portfolio
4. Reflection

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Discovery Project Overview

1. Students worked in groups of 3-4
2. All groups started with the same assignment.
3. Students used the OEIS to test their conjectured sequence
4. Each group then used the OEIS to identify an interesting closed formula, recurrence, connection to another mathematical problem.
5. I guided students from here based on their responses to further their research.
6. Required groups meet with me at least two times to check in, and gave a third optional meeting time.

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Discovery, Part I

1. **Read article on OEIS** and reflect on
   a. when it is useful to consult the OEIS
   b. what information the OEIS provides
2. **Investigate the question:**
   a. For n = 1, 2, … , 5 make these computations:
      i. Find partitions (e.g. {3, 2+1, 1+1+1})
      ii. Take products of parts (e.g. {3,2,1})
      iii. Find the maximum of the previous set (e.g. 3)
      iv. Enter resulting sequence into OEIS
      v. Verify by calculating next value of *n*
      vi. Repeat as necessary until OEIS gives you your sequence.

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Course-Embedded Undergraduate Research Experience (CURE)

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Discovery, Part 1-2

3. **Use the OEIS to**
    a. Determine if there is a closed formula, recursive sequence, etc.
    b. Are there other mathematical problems for which this sequence occurs?
4. **Based off their interest groups continued to**
    a. prove a closed formula, recursive formula
    b. Make connections
        i. Maximal size of an Abelian subgroup of S_n
        ii. How many 1s are needed to represent *n*
        iii. Keyboard problem
        iv. Maximum number of maximal cliques in a graph with *n* vertices

Discovery, Final Product

5. **Communicate your results**
    a. Prepare a Group poster
    b. Write an individual summary

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